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# UNCERTAINLY MEASUREMENT

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In recent years, a series of metrics began to develop that allow the quantification of specific properties of process models. These characteristics are, for example, complexity, comprehensibility, maintainability, cohesion and uncertainty. This work is focused on defining a method that allows to measure the uncertainty of process models that was modelled by Stochastic Petri Nets (SPN). Principle of this method consists in mapping the set of all reachable marking of SPN into the continuous-time Markov chain and then calculating its steady-state probabilities. The uncertainty is then measured as the Shannon entropy of the Markov chain (it is possible to calculate the uncertainty of the specific subset of places as well as whole Petri net). Alternatively, the uncertainty is quantified as
a percentage of the calculated entropy against maximum entropy.

## 1. Introduction and related works

It has been known for long time that within development, the change of processes are uncertain and interconnected (Hirschman, 1967; Simon, 1972; Brinkerhoff and Ingle, 1989). Complexity and uncertainty have become critical issue for modelling applications, opening new ways for the use and development of models. Increasingly models are being recognised as essential tools to learn, communicate, explore and resolve the particulars of complex, for example environmental, problems (Sterman, 2002; Van den Belt, 2004, Brugnach 2008). However, this shift in the way in which models are use has not always been accompanied by a concomitant shift in the way in which models are conceived and implemented. Too often, models are conceived and built as predictive devices, aimed at capturing single, best, objective explanations. Considerations of uncertainty are often downplay and even eliminated because it interfered with the modelling goals. When modelling and analysing business processes, the main emphasis is usually on the validity and accuracy of the model, that means, the model meets the formal specification and also models the correct system. In recent years, a number of measures have begun to develop, enabling quantification of the specific features of process models. These characteristics are, for example, complexity, comprehensibility, maintainability, coherence, and uncertainty. The work is aimed at defining a method that allows to measure the uncertainty of process models that was modelled using the stochastic Petri nets (SPN). The principle of this method consists of mapping the reachable SPN markings into a continuous Markov chain, and then calculating the stationary probabilities of markings. Uncertainty is then measured as the entropy of the Markov chain (it is possible to calculate the uncertainty of a specific subset of sites as well as the entire network). Alternatively, the uncertainty index is quantified as a percentage of the calculated entropy versus the maximum entropy (the resulting value is normalized to the interval <0.1>). Calculated entropy can also be used as a measure of model complexity (Ibl and Čapek 2016).

Uncertainty

A realistic modelling and simulation of complex systems must include the nondeterministic features of the system and the environment. By 'nondeterministic' we mean that the response of the system is not precisely predictable because of the existence of uncertainty in the system or the environment, or human interactions with the system (Oberman 2001). Fig.1 shows relationship between uncertainty, data and model.

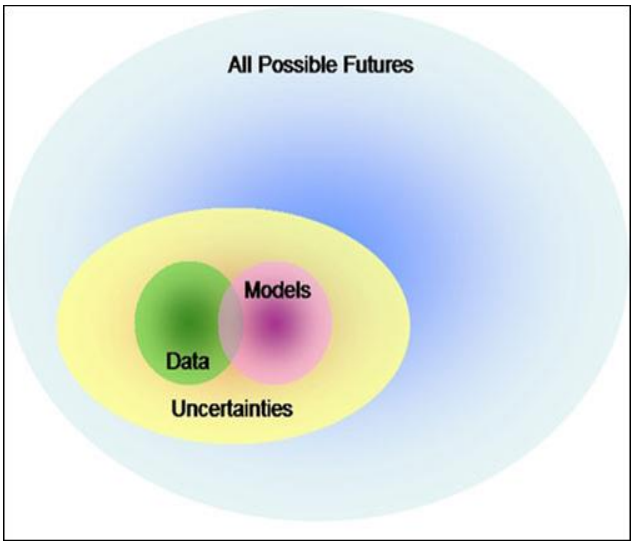

Fig.1 Uncertainties, Data and Models (according Carpertner (2006))

In a measurement, the uncertainty is quantified as a doubt about the result of the measurement. Measurement device outputs are data displaying information about the measured quantity. Entropy (or uncertainty) and information, are perhaps the most fundamental quantitative measures in cybernetics, extending the more qualitative concepts of variety and constraint to the probabilistic domain. Variety and constraint, the basic concepts of cybernetics, can be measured in a more general form by introducing probabilities. Assume that we do not know the precise states of a system, but only the probability distribution $P(s)$. Variety $V$ can be then expressed as the Shannon entropy $H$:

$$H(P) = -\sum_{s\in S} P(s).\log P(s)$$

$H$ reaches its maximum value if all states are equiprobable, that is, if we have no indication whatsoever to assume that one state is more probable than another state. Like variety, $H$ expresses our uncertainty or ignorance about the system's state. It is clear that $H = 0$, if and only if the probability of a certain state is equal to 1 (and all other states are equal to 0). In that case, we have maximal certainty or complete information about what state the system is in. We define constraint that reduces uncertainty, i.e. the difference between maximal and actual uncertainty. This difference can also be interpreted in a different way, as information. Indeed, if we get some information about the state of the system (e.g. through observation), then this will reduce our uncertainty about the system's state, by excluding or reducing the probability of a number of states. The information we receive from an observation is equal to the degree to which uncertainty is reduced.

For uncertainty identification is possible to use the Ishikava fishbone diagram, see Fig. 2.

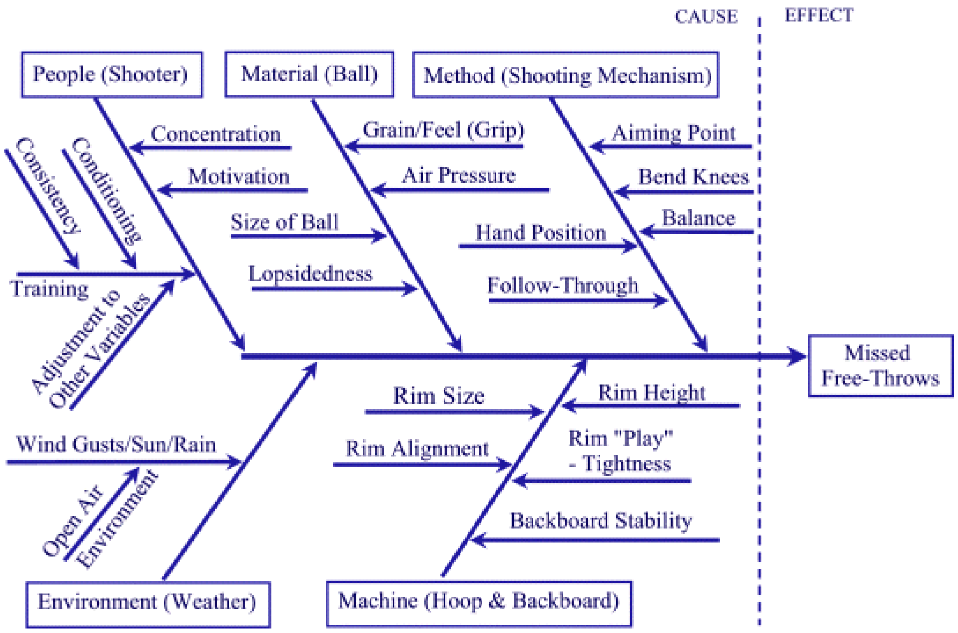

Fig. 2 Fishbone diagram (Source: MoreSteam (2013))

Dr. Kaoru Ishikawa developed the “Fishbone Diagram” at the University of Tokyo in 1943. Hence, the Fishbone Diagram is frequently referred to as an "Ishikawa Diagram” **The diagram is used in process improvement methods to identify all of the contributing root causes likely to be causing a problem**. The Fishbone diagram is an initial step in the screening process. After identifying potential root cause(s), further testing will be necessary to confirm the true root cause(s). This methodology can be used on any type of problem, and can be tailored by the user to fit the circumstances. Ishikawa, K., (1989). The example we have chosen to illustrate is "Missed Free Throws" (the one team lost an outdoor three-on-three basketball tournament due to missed free throws) MoreSteam (2013). In manufacturing settings, the categories are often: Machine, Method, Materials, Measurement, People, and Environment. In service settings, Machine and Method are often replaced by Policies (high-level decision rules), and Procedures (specific tasks).

## 2. Petri net

A gentle introduction into Petri nets modelling approach is made for example by WoPeD (WoPeD 2005) where Petri nets are described as follows: “**Petri Nets** are a graphical and mathematical modelling notation first introduced by Carl Adam Petri's dissertation published in 1962 at the Technical University Darmstadt (Germany). A Petri Net consists of **places**, **transitions**, and **arcs** that connect them. Places are drawn as circles, transitions as rectangles and arcs as arrows. Input arcs connect places with transitions, output arcs connect transitions with places. Places are passive components and model the system state. They can contain **tokens**, depicted as black dots or numbers. The current state of the Petri Net (also called the **marking**) is given by the number of tokens at each place. Transitions are active components that model activities that can **occur** and cause a change of the state by a new assignment of tokens to places. Transitions are only allowed to occur if they are **enabled**, which means that there is at least one token on each input place. By occurring, the transition removes a token from each input place and adds a token to each output place. Due to their graphical nature, Petri Nets can be used as a visualization technique like flow charts or block diagrams but with much more scope on concurrency aspects. As a strict mathematical notation, it is possible to apply formal concepts like linear algebraic equations or probability theory for investigating the behaviour of the modelled system. A large number of software tools were developed to apply these techniques.

Examples of properties that are widely verified on Petri's networks are liveness, boundedness, reachability, fairness, and others. Verification of individual properties may be analytical (for basic classes of Petri nets) or have simulation character (for higher classes of Petri nets). The other way of development was to broaden the basic definition of the Petri nets so that their modelling power complies with specific requirements. Examples include timed and stochastic Petri nets, which allow refinement of individual states changes with deterministic (Dorda 2008, Zuberek, 1991, Holliday and Vernon, 1987) or stochastic (Ajmone Marsan, 1990)time considerations.

## 3. Example

As an example,according (Ibl and Čapek 2016), is presented a stochastic Petri net consists of 5 places and 5 transitions, see Fig. 3. The model contains the essential characteristic features that are included in the process model. These elements are, for example, the sequence (e.g., transition T4), AND-split (transition T1), AND-join (transition T6), XOR (transitions T6 and T5 or T6 and T3).

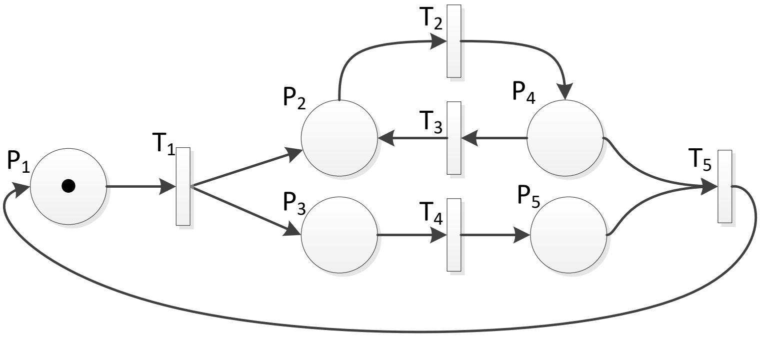

Fig. 3 Example of a stochastic Petri net

The set of all reachable markings $R(M_0)$ of this example Petri net contains 5 markings:

| | $M_0$ | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|---|
| $p_1$ | 1 | 0 | 0 | 0 | 0 |
| $p_2$ | 0 | 1 | 0 | 1 | 0 |
| $p_3$ | 0 | 1 | 1 | 0 | 0 |
| $p_4$ | 0 | 0 | 1 | 0 | 1 |
| $p_5$ | 0 | 0 | 0 | 1 | 1 |

With consideration of transition firing rates, for example, $\Lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$ , the given net is shown in Fig. 4 as a Markov chain.

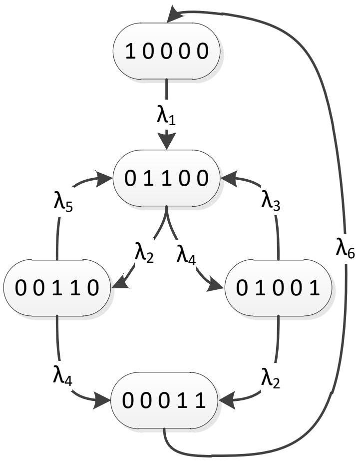

Fig. 4 Corresponding Markov chain

The solution of this chain, for $\Lambda = (5,2,3,3,2,1)$ is stationary probability vector:

$$\eta = \begin{bmatrix} 0.0385 \\ 0.2692 \\ 0.1538 \\ 0.3462 \\ 0.1923 \end{bmatrix}$$

The entropy of the example network can then be expressed by:

$$\begin{aligned} H(SPN) = -(&0.0385 \log_2 0.0385 \\ &+ 0.2692 \log_2 0.2692 + 0.1538 \log_2 0.1538 \\ &+ 0.3462 \log_2 0.3462 + 0.1923 \log_2 0.1923) = 2.093 \end{aligned}$$

Reference limit (maximum entropy) is in this case is $\log_2 5 = -2.3219$. The uncertainty for this particular case is determined by the relation $-H(SPN)/\log_2 |R(M_0)|$, i.e. $2.093/2.3219 = 0.9015$. This result can be loosely interpreted as the fact that the uncertainty of the example stochastic Petri net (SPN) reaches 90.15%.

Uncertainty can be then analysed as a response to changes in a parameter of SPN, for example, the number of tokens in the initial marking or an adjustment of a specific parameter $\lambda \in \Lambda$. In the following is presented an example that shows the development of the uncertainty with a different initial marking. Fig. 5 indicates that the increasing number of tokens in the initial marking (in the place p1) decreases the uncertainty of SPN.

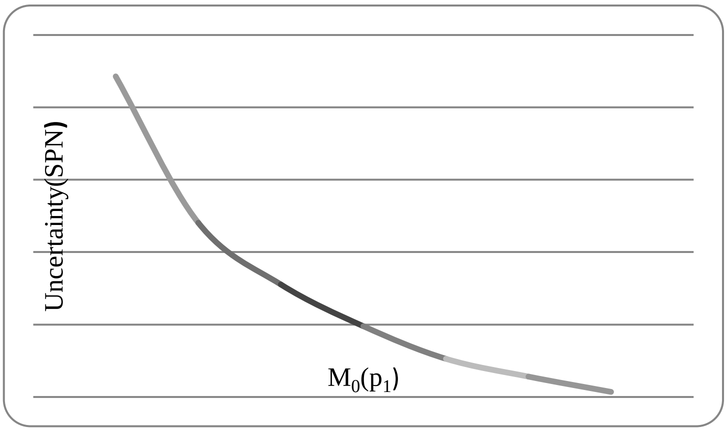

Fig. 5 Uncertainty vs. number of tokens

## 4. Conclusion

Measurement of uncertainty can be an appropriate tool for assessing the relevance and the predictability of process models, and thus serve to more effective managerial decision making. The degree of uncertainty in the process model is directly dependent on two main indicators. The first is the number, the ratio and the distribution of specific elements (OR, XOR, AND, and LOOP) in the model. These elements provide branching, synchronization and cycles in the model, and thus are the main building blocks of process models that shape its specific structure. One of other approaches to the measurement of uncertainty in the process model (Jung et al., 2011)is based on quantifying the entropy of partial substructures of the model at different levels of abstraction. However, this approach takes into account only static structure of the model and does not take into account dynamic component, which can be expressed in Petri nets using tokens.

Keywords: **uncertainty, entropy, modelling, stochastic Petri nets**

## References

[1] Ajmone Marsan M., Stochastic Petri nets: an elementary introduction. [in:] Advances in Petri nets, ed. Rozenberg G., Springer-Verlag, New York 1989.

[2] Brinkerhoff D., Ingle M., Integrating blueprint and process: a structured flexibility approach to development management, Public Administration and Development, 9(5), 1989, 487-503.

[3] Brugnach M., et al., Complexity and uncertainty: rethinking the modelling activity. [in:] Environmental modelling, software and decision support: state of the art and new perspectives, ed. A. J. Jakeman A. J., Voinov A. A., Rizzoli A. E., Chen S. H. Elsevier, Amsterdam 2008.

[4] Carpenter S. R., et al., Ecosystems and human well-being: scenarios, Millennium Ecosystem Assessment - MA (2005) project (eds.), Island Press, Washington 2006.

[5] Dorda M., Introduction into Petri nets Retrieved from http://homel.vsb.cz/~dor028/Nekonvencni_metody_1.pdf, 2008.

[6] Hirschman A., The principle of the hiding hand, National Affairs, Issue 6, 1967.

[7] Holliday M. A., Vernon M. K., A Generalized Timed Petri Net Model for Performance Analysis. IEEE Transactions on Software Engineering, SE-13**,** 1987, 1297-1310.

[8] Ibl M., Čapek J., Measure of Uncertainty in Process Models Using Stochastic Petri Nets and Shannon Entropy. Entropy, 2016, 18**,** 33.

[9] Ishikawa K., Introduction to Quality Control. Published by JUSE Press Ltd. Softcover reprint of the hardcover 1st edition 1989 Distributed outside Japan and North America by: Chapman & Hall 2 -6 Boundary Row, London.

[10] Jung Y.-Y., Chin C.-H., Cardoso J., An entropy-based uncertainty measure of process models. Information Processing Letters, 2011, 111, 135-141.

[11] Moresteam, Retrived from https://www.moresteam.com/toolbox/fishbone-diagram.cfm, 2013.

[12] Morgan D., Retrived from http://bvcentre.ca/files/research_reports/09-12-Morgan-Complexity-Uncertainty.pdf, 2010.

[13] Oberman W.L., et al., Error and uncertainty in modeling and simulation. Reliability Engineering and System Safety 75, 2002, 333-357.

[14] Root H., et al., Managing complexity and uncertainty in development policy. Retrieved from https://www.researchgate.net/profile/Hilton_Root/publication/280884255_Managing_